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SPACE CHARGE EFFECTS ON SATELLITE ELECTRIC FIELD MEASUREMENTS

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Current biased probes have been previously employed on satellites to make electric field measurements. These probes emit electrons through sunlight induced photoemission and collect both ions and electrons from nearby plasmas. Under certain conditions, the photoemission current is orders of magnitude greater than the collected currents. When, in addition, plasma gradients are present, electric charge accumulation, or "space charge", can affect the accuracy of the electric field measurements. To obtain a better understanding of these space charge effects, we have conducted a laboratory investigation of the characteristics of an emitting probe in a vacuum and in a low density plasma. The results are discussed in this paper.

INTRODUCTION

"Space charge" refers to the negative charge that often accumulates in the space near an electron emitter. "Space charge effects" that are of specific interest to this work are the tendency for space charge to repel emitted electrons and impede the emitted electron current flow away from the electron emitter.

As an example of how these space charge effects can be relevant to satellite probes, consider two strongly photoemitting probes on a satellite. If conditions are identical at the probes, then the currents and voltages of the two probes should be equal. The currents and/or voltages are not equal, however, if space charge effects on the two probes are not equal. Because the emitted electron current which escapes and does not return to a probe must overcome the repulsion from space charge, if the currents of the two probes are equal, then the voltage of the probe which experiences greater space charge effects must be lower than the voltage of the other probe. Conversely, if the voltages of the probes are the same, then emitted electron current which escapes and does not return to probe which experiences the greater space charge effects is less than the corresponding current of the other probe. Hence, even if all other conditions are identical, if the space charge effects on the two probes are not identical, then the currents and/or voltages of the two probes are not equal. Specifically, all other things being equal, the current and/or voltage of the probe which experiences more space charge effects are/is lower than that of the other probe.

Satellite probes are often used to make electric field measurements in the following manner. Two probes are placed 180° apart, each at the end of a

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long boom. The probes are conductors which are electrically isolated from the booms. They collect both ions and electrons from nearby plasma and photoemit electrons into nearby plasma. Both probes are biased to the same current, that is, equal values of current are made to pass through each probe. The difference between the probe voltages divided by the distance between the probes is taken to be a measure of the average electric field in the nearby plasma.

Relatively small amounts of plasma can have rather large effects on space charge. As an electron emitter collects ions from the plasma, the conservation of angular momentum allows ions to spend long periods of time in the vicinity of the electron emitter, where they alleviate the negative space charge from the emitted electrons. Hence, it is possible for plasma gradients to indirectly cause errors in electric field measurements by causing differences in the space charge effects at the two probes being used to make the electric field measurements.

THEORY

It can be shown (see Appendix A for details) that when emitted electron temperature, magnetic field, and leakage current effects are neglected the error in electric field measurements E_E (in units of Volts/meter) made by two cylindrical probes due to space charge effects in the presence of plasma gradients is

$$E_E \leq [(r\beta^2/L)(I/14.68)]^{2/3}/d \qquad \text{Eq. *4/20,5}$$

where I is in units of 10^{-6} amps and is the fraction of the current bias which is emitted current rather than collected current (see the last paragraph of Appendix A for more details on I), L is the length of the probes, r is the radius of the sheath around each probe, d is the distance between the probes (in units of meters), and β is a constant which depends on the ratio of the probe sheath and probe radii and in practice has a value close to 1.0 under most circumstances.

Some "typical" values which can be applied to Eq. *4/20,5 are $I = 1 \times 10^{-6}$ Amps, $r \simeq \lambda_D$ (a Debye length) $\simeq 10$ m, and $L = 10$ m. Substituting these values into Eq. *4/20,5 yields $E_E d \leq 0.17$ Volts, where $E_E d$ is the difference in voltage between the probes due to space charge effects. Note that $I \geq 0$ and $E_E > 0$, which means that space charge adds to an electric field measurement an erroneous electric field that is antiparallel to the density gradient (again see Appendix A for details).

For a given probe radius and length, eq. *4/20,5 suggests that minimizing I will minimizing E_E, that floating the probes (i.e., setting I to 0) best minimizes space charge errors. Unfortunately, probe temporal response can be quiet long for strongly emitting floating probes. Hence, to minimize space charge effects, I should be made as small as possible without an undue compromise of the temporal response of the probes.

Also note that eq. *4/20,5 suggests that maximizing L will also minimize E_E. However, I is usually proportional to L; so increasing L will not help in general as I also increases, and decreasing I will not help if it is done by decreasing L. I is also usually proportional to the radius of the probe r_p and so minimizing r_p will clearly help. There are of course limits as to how small r_p can be made, namely, a limit as to how thin a probe can be made and a limit as to how small the current bias can be made.

Not all satellite electric field probes are cylindrical; many, if not most, are spherical. It can be shown (see Appendix A for details) that the error in electric field measurements made by two spherical probes due to space charge effects in the presence of plasma gradients is

$$E_E \leq [\alpha^2 I/29.36]^{2/3}/d \qquad \text{Eq. *4/20,8}$$

where I is in units of 10^{-6} amps and is the fraction of the current bias which is emitted current rather than collected current, d is the distance between the probes (in units of meters), and α is a function of the radii of the anode and emitting cathode. Some typical values of α^2 are 0.509, 1.022, 2.073, 4.002, and 8.523 which correspond to values of the ratio of anode radius to cathode radius of 1.8, 4.4, 14, 160, and 100000, respectively. For $I = 1 \times 10^{-6}$ Amps and $\alpha^2 \approx 4$, Eq. *4/20,8 yields $E_E d \leq 0.27$ Volts.

Eqs. *4/20,5 and *4/20,8 point out some similarities and differences between cylindrical and spherical probes. In both geometries, making I small is important if E_E is to be kept to a minimum and limitations on the minimum probe size and the minimum current to which the probes can be biased also limit how small I can be made. On the other hand, although each situation in which probes are used must be considered separately, cylindrical geometry has the seeming advantage that E_E can also be reduced by making L large (while keeping I constant by making r_p small).

EXPERIMENTAL METHODS

The experiment was carried out in a cylindrical stainless steel chamber of length 64 cm and radius 30 cm. Base pressure was ~ 3×10^{-7} torr. An emissive probe of length 55 cm and diameter 5×10^{-3} inch was placed in the center of the chamber. The emissive probe was heated into thermionic emission by a variable, half-wave rectified, 60 Hz heating voltage. Voltage bias sweeps between -10V and 5V were applied to the probe though a CA3140E operational amplifier that was configured as a voltage follower. An ORTEL Brookdeal 9415 linear gate was employed to take data during the off cycle of the heating voltage.

Plasma could be created by bleeding air into the machine until the pressure reached the 10^{-3} torr range. The voltage drop across the probe during the on part of the heating cycle was large enough that some of the electrons were emitted with enough energy to ionize. Once plasma had been created in the 10^{-3} torr range, plasma could be produced at lower pressure. The plasma density was roughly proportional to the neutral pressure.

Three problems made the measurements difficult. The origin of all three problems was the finite impedance between the primary and secondary of the heating voltage transformer. The currents that flowed through the transformer's finite impedance and also flowed through our measuring apparatus and thus were a source of error. The three problems and our solutions to them are discussed in detail in Appendix B.

Results and Discussion

The laboratory data shown in Fig. *8/8,1 demonstrate of some of the possible errors in electric field measurements made by strongly emitting probes due to space charge effects in the presence of plasma gradients. In Fig. *8/8,1, electron current that is emitted from the probe and does not return to the probe is shown as positive current, while electron current that is collected by the probe is shown as negative current. The current vs. voltage characteristics A and B shown in Fig. *8/8,1 are characteristics of the same laboratory probe taken under different probe and plasma conditions. Specifically, probe emission was stronger and the plasma more dense when characteristic A was taken.

In the next paragraph, we will assume that characteristics A and B are the characteristics of two probes on a satellite that are being used to make electric field measurements. This assumption will allow us to make several comments with regard to possible errors in satellite electric field measurements. It would of course be an improbable coincidence for the characteristics of any two probes on any satellite to be identical to characteristics A and B. There is reason to believe, however, that when space probes are in plasma gradients, it is possible for the space probe characteristics to have the same features as those of characteristics A and B that cause errors. More will be said about this later in this section.

So, let us suppose for the moment that characteristics A and B are the characteristics of two probes on a satellite that are being used to make electric field measurements. Let us further assume that both probes are biased to the same current (that is, that there is circuitry on board the satellite that changes the voltage of each probe until the net emitted current which flows through each probe is equal to the desired bias current) and that the voltage difference between the two probes is measured. The standard method for calculating the average electric field between two current biased probes on a satellite is to divide the measured voltage difference between the probes by the distance between the probes.

Using this standard method, the electric field calculated from characteristics A and B depends on the current bias that is chosen. If the probes are biased at a current of 12 μA, the probes will indicate an electric field that points from probe A towards probe B. If the probes are biased at approximately 9 μA, the voltage difference, and hence the calculated electric field also, between the two probes is approximately half the value it is for a current bias of 12 μA. If the current bias is chosen to be approximately 6 μA, then the probes will indicate that there is very little or approximately zero electric field. And, if a current bias of between 0 and 5 μA is chosen, the probes will indicate an electric field that points, not from probe A towards probe B, but from probe B towards probe A. Hence, the magnitude and even the direction of the calculated electric field will depend on the current bias that is chosen.

It is disturbing that the electric field calculated by this standard method should depend on the current bias that is chosen. Satellite probes are usually biased at "intermediate" currents, that is, currents that are not close to the saturation currents. And, it is often assumed that all intermediate current biases will yield the same electric field measurement. Yet, characteristics A and B suggest otherwise. Characteristics A and B suggest that if the characteristics of two satellite probes are similar to characteristics A and B, then the electric field measured by the satellite probes is either positive, zero, or negative depending on the current bias that is chosen.

Although characteristics A and B are alike in many ways, they are not identical. In particular, at intermediate currents the slope of characteristic A is steeper than the slope of characteristic B. It is this difference in slopes that allows the voltage difference between characteristics A and B to vary with current. Hence, the features of the characteristics which lead to the errors described are the slopes of the characteristics at intermediate currents, slopes which are not equal to one another.

Space charge effects are responsible for the difference in the slopes of characteristics A and B at intermediate currents and are therefore also responsible for the errors in electric field measurements discribed above. Specifically, at intermediate currents the slope of characteristic A is steeper than the slope of characteristic B because, as will be shown shortly, characteristic A does not noticeable space charge effects while characteristic B does.

As previously noted, the presence of ions near an electron emitting probe reduces the currrent limiting space charge effects on that probe. Furthermore, the saturated collected electron current of characteristic A is greater than that of characteristic B, which indicates that the plasma near the probe was more dense when characteristic A was taken than it was when characteristic B was taken. Hence, we might expect that the emitted current of characteristic A is less space charge limited than that of characteristic B; and this is indeed the case.

The slope of characteristic A at intermediate currents is fairly well described by emitted electron current that is not space charge limited. If the current emitted from the probe I_e is not is not limited by space charge and can be described by a temperature T, then the current I that flows from the probe to the plasma is describe by

$I = I_e$ when $V_p - \phi_p \leq 0$ Eq. *8/8,1

and $I = I_e \exp[-e(V_p - \phi_p)/T]$ when $V_p - \phi_p > 0$ Eq. *8/8,2

where V_p is the voltage of the probe, ϕ_p is the potential of the plasma near the probe, the temperature T is in units of eV, and the collected currents have been ignored. In Fig. *8/8,2, Eqs. *8/8,1 and *8/8,2 are shown as characteristic C. The characteristic A of Fig. *8/8,2 is the same as characteristic A of Fig. *8/8,1. As can be seen in Fig. 8/8,2, characteristic A is in fairly good agreement with characteristic C at intermediate currents and we conclude from this that the probe was not space charge current limited when characteristic A was measured.

****values of T and ϕ_p****

On the other hand, characteristic B of Fig. *8/8,1 is somewhat, but not completely space charge current limited, that is, it appears that there were some ions present when characteristic B was measured but not enough to completely negate the effect of negative charge accumulation near the electron emitting probe. When a characteristic is completely space charge current limited it is described by (see Appendix A, Eqs. *4/19,1 and 4/19,3)

$$I = I_e \qquad \text{when } V_p - V_B \leq -[(r\beta^2/L)(I/14.68)]^{2/3} \qquad \text{Eq.*4/19,1}$$

$$\text{and } V_p - V_B = (T/e)\ln(I_e/I) - [(r\beta^2/L)(I/14.68)]^{2/3}$$

$$\text{when } V_p - V_B > -[(r\beta^2/L)(I/14.68)]^{2/3},\ I < I_e \qquad \text{Eq. *4/19,3}$$

where r is the anode radius, β is a constant which depends on the ratio of the anode and emitting cathode radii and in practice has a value close to 1.0 under most circumstances, L is the length of the cathode, and I is the current that flows from the probe to the plasma and is in units of $X10^{-6}$ Amps. In Fig. *8/8,3: Eqs. *4/19,1 and *4/19,3 are shown as characteristic D; Characteristic B is the same as characteristic B of Fig. *8/8,1; and Eqs. *8/8,1 and *8/8,2 are shown as characteristic E (for both characteristics D and E, *****the values of T and ϕ_p are *** and ***, respectively). Note that charactertistic B lies between characteristic D, which it would be in good agreement with if the probe were completely space charge current limited, and characteristic E, which it would be in good agreement with if the were no space charge effects. Hence, characteristic B is partially space charge current limited.

The difference in voltage at a given current between characteristics D and E is proportional to error in electric field measurements due to space charge when emitted electron temperature, magnetic field, and leakage current effects on space charge can be neglected. Specifically, the voltage difference between characteristics D and E is equal to $E_E d$ where E_E, which is given by Eq. *4/20,5, is the electric field measurement error and d is the distance between probes. As was noted in the theory section, E_E goes to zero as the current bias goes to zero. Therefore, the true electric field corresponding to characteristics A and B points from B to A.

Emitted electron temperature effects are expected to make this difference in voltage, the error, less[1]. Magnetic field effects on the other hand will tend to make it more difficult for emitted electrons to escape from the probe and will hence accentuate space charge effects and make the error worse (the magnetic field will constrict the electron flow to along field lines and the charge accumulation will be greater along the constricted flow).

The effects of magnetic fields are shown in Fig. *8/8,4. Eqs. *8/8,1 and *8/8,2 are shown as characteristic F in Fig. *8/8,4 and Eqs. *4/19,1 and

1.Langmuir,I., and Blodgett,K., Phys. Rev. 22, 347 (1923)

*4/19,3 are shown as characteristic G. For both characteristics F and G, the values of T and ϕ_p are 0.3 eV and 0.8 V, respectively. Characteristic H shown in Fig. *8/8,4 was measured when no plasma was present, when the probe was in a vacuum. As can be seen, there is fairly good agreement between characteristics H and G, especially at low currents. Only the earth's magnetic field (approximately half a gauss in our laboratory) was present when characteristic H was measured. In this magnetic field, the ratio of emitted electron gyroradius to the vacuum chamber radius is about 10. Hence, it is not surprising that the earth's magnetic field seems to have accentuated the space charge effects, that characteristic H lies to the left of characteristic G in Fig. *8/8,4.

The accentuation of space charge effects by magnetic field is shown more clearly by characteristic I in Fig. *8/9,4. Characteristic I was also measured when the probe was in a vacuum. But when characteristic I was measured, there was also a stronger magnetic field present; rows of permanent magnets had been placed around the vacuum chamber and the magnetic field in the vacuum chamber ranged from about 2 gauss on axis where the probe was to about 40 gauss near the magnets.

Leakage current effects on space charge are not as well understood, but it is known that leakage currents can have pronounced and undesirable effects on electric field measurements when precautions are not taken. The authors feel that leakage current effects on space charge, and the subsequent effects on electric field measurements, warrant further investigation.

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Appendix B

The measuring of probe current vs. voltage characteristics was complicated by the finite impedance between the primary and secondary of the heating voltage transformer. Fig. *7/25,1 is a schematic of the electrical components that are pertinent to this discussion. Components A through C provide the voltage that heats the probe D into thermionic emission. Component A represents the wall voltage (120 V, 60 Hz), B is a variac or variable voltage transformer, and C is an isolation transformer. Component E is a voltage ramp (that is, a voltage with respect to ground that increases linearly with time) which is applied to the probe through the voltage follower F. G is the point at which the "probe voltage" or "bias voltage of the probe" is measured. Component H represents the vacuum vessel wall and its ground. I depicts the vacuum or plasma that lies between the probe and vacuum vessel wall.

Note that there are four grounds, M, H, K, and N. The current that we are interested in, the current which is suppose to correspond to the current in a probe's current vs. voltage characteristic, is the current that flows between the probe D and the ground H. The impedance between the probe D and the ground N should be quite large; it should be on the order of the input resistance of the voltage follower F, which is a CA3140 op amp and has a nominal input resistance of $10^{12}\,\Omega$. Ideally the impedance between the probe D and ground M is also quite large because of the large impedance between the primary and secondary of the isolation transformer C. In this case, the current that flows from ground H to probe D completes its circuit by flowing from probe D through the voltage follower F and the resistor J to ground K. The current through the probe is then calculated by dividing the voltage measured at point L by the resistance of J.

Unfortunately, the impedance between the primary and secondary of our isolation transformer C is small enough to allow significant current to flow between ground M and grounds H and K. In fact, during most times in the heating cycle, the current flowing to our ground K is dominated by current flowing from M through C, F, and J to K and not by the current that we wish to measure which is the current that flows from H through I, D, F, and J to K.

There are two types of impedance across C that are of concern, capacitance and resistance. There is finite capacitance between the primary and secondary windings of C because of their proximity to one another. Fig. *7/25,2 is a simplified approximation of Fig. *7/25,1 and depicts the circuit elements essential in understanding how we avoid the problem caused by the capacitance across the isolation transformer. V_p in Fig. *7/25,2 corresponds to the voltage across the primary of the isolation transformer and Z_c is the value of the impedance due to the finite capacitance across the transformer. E is a voltage sweep of the same value as the voltage sweep depicted in Fig. *7/25,1 and represents the output of the voltage follower F, R_J is the resistance of the resistor J, and R_s is the "sheath" resistance or the approximate resistance between the probe D and the ground H.

It is common practice to make R_J much less than R_s, and Z_c is usually much greater than either R_J or R_s. In other words,

$Z_c >> R_J >> R_s$ *Eq. 7,25,1

Hence, most of the current that flows from ground M flows to ground K and not to ground H. Also, the phase of this cyclical current proceeds V_p by $\pi/4$; the current passes through zero when V_p passes through its maximum and minimum. We can essentially eliminate the effect of this unwanted current on our measurements of probe current vs. voltage characteristics by monitoring V_p and triggering a temporally short voltage pulse at the minimum of each cycle of V_p. This voltage pulse is in turn used to trigger a linear gate, which samples the voltage at point L in fig. *7/25,1 during (and only during) the voltage pulse. In this way, data are sampled periodically at a time in the cycle of the unwanted current when the amplitude of the current is near zero.

There is also a finite resistive impedance across the isolation transformer C. The resistance across our particular transformer only becomes troublesome when the heating voltage is applied to the probe for over 20 seconds. It is our conjecture that with time the transformer becomes warm, the resistance across C decreases (perhaps due to a low melting point of some material(s) in the insulation between the primary and secondary), and as a result the current across the resistance increases and starts to be noticeable on the measured current vs. voltage characteristic.

Our solution to this problem is to simply measure the characteristics quickly, before there is a noticeable resistive current. There are of course limits as to how quickly the characteristics can be measured. We've noticed that the emitted current continues to increase for the first few seconds after the heating voltage is applied before leveling off to a constant value. Presumably, the probe is warming up during these first few seconds. Note that the warming takes place immediately after the heating voltage is applied to the probe while the resistance does not become too small for making measurements until some time after. Further, these two currents are of opposite sign, that is, warming increases the emitted current while the finite resistance decreases the measured emitted current (because some of the emitted current flows through ground M rather than ground K). Hence, we have a check on these two effects. We measure each characteristic twice, with enough time between the measurements to allow the transformer to cool down. The first characteristic is measured by sweeping the probe from low voltage to high voltage; the second characteristic is measured by sweeping from high to low. The measurement is good when there is good agreement between the two characteristics. When the warming of the probe and/or the warming of the transformer is a problem, the two characteristics do not agree at either low and/or high voltages, that is, at either early and/or late times in the sweeps.

Lastly, it should be noted that although increasing the size of the transformer would probably increase the time until the transformer warms up and develops a small resistance between the primary and secondary, increasing the size would probably also increase the capacitance between the primary and secondary. An increase in this capacitance would make the problem of finite capacitive impedance more acute.

APPENDIX A

Child [Child, Phys. Rev., 32, 492(1911)] and Langmuir [Langmuir,I., Phys. Rev., 2, 450 (1913)] were the first to realize that the maximum amount of current that can flow between an electron emitting cathode and a nonemitting anode in a vacuum is limited by space charge, that is, by the electrical charge of the electrons flowing in the space between the cathode and anode. The electrons take a finite amount of time to travel from cathode to anode and the charge of those emitted first tends to impede the flow of those emitted later. When emission from the cathode becomes large, a minimum in electric potential forms near the cathode and some of the low energy emitted electrons return to the probe, net current is reduced.

It has been known for some time [Kingdon, K.H., Phys. Rev., 21, Series II, 408 (1923)] that small amounts of plasma can greatly affect the space charge near an electron emitter. This is because the conservation of angular momentum allows ions from the plasma to spend long periods of time in the vicinity of the probe, where they alleviate the negative space charge from the emitted electrons, before being collected by the probe.

Quantifying the effect that ions have on space charge can be rather difficult because it depends on, among others, such things as the ion velocity distribution and the mechanisms by which the ions can lose angular momentum and thereby eventually collide with the probe. Nevertheless, in general, the more ions there are present, the less space charge impedes current flow. We believe that space charge effects give the maximum error in electric field measurements when one probe is in a vacuum (no ions, maximum reduction of current by space charge) and the other probe is in a dense plasma (many ions, no reduction of current by space charge). This is the situation which we consider in this section. The reader should note that we will not take into account the effects on space charge of emitted electron temperature, magnetic field, and leakage current, that is, current which flows directly between the probes and the satellite without passing through the ambient region through which the satellite is passing and in which one is trying to measure the electric field.

Let us first think about the probe in the plasma that is dense enough that there is no reduction in current due to space charge (and is still tenuous enough that the emitted current is much greater than the collected current). If the current emitted from the probe I_e can be described by a temperature T, then the current I that flows from the probe to the plasma is described by

$I = I_e$ when $V_p - \phi_p \leq 0$ Eq. *4/9,1

and $I = I_e \exp[-e(V_p - \phi_p)/T]$ when $V_p - \phi_p > 0$ Eq. *4/9,2

where V_p is the voltage of the probe, ϕ_p is the potential of the plasma near the probe, and <u>the collected currents have been set to zero and ignored</u>. Note that another form of eq. *4/9,2, a form which will be more useful later, is

$V_p - \phi_p = (T/e)\ln(I_e/I)$ when $V_p - \phi_p > 0$, $I < I_e$ Eq. *4/20,1

We begin our contemplation of the other probe, the probe in a vacuum, by defining nomenclature. The current that is emitted from the probe will again be denoted by I_e and the current that flows between the probe and the vacuum boundary will be called I. The reduction in current that occurs if there is a minimum in potential near the probe will be named I_R and the sign of I_R will be such that $I \equiv I_e + I_R$. The electric field at the probe will be called E_o and will be of such sign that when it points towards the probe $E_o < 0$. When $E_o > 0$, there is a minimum in potential near the probe and this minimum will be known as the "virtual cathode". The Child-Langmuir law [Child, Phys. Rev., 32, 492(1911); Langmuir,I., Phys. Rev., 2, 450 (1913)], which is useful when considering this probe, relates the difference in voltage between a nonemitting anode and an electron emitting cathode in a vacuum to the current that flows between them when the electric field at the cathode is zero. Since the electric field at a virtual cathode is zero, the Child-Langmuir law can be applied to an anode and virtual cathode. Hence, the Child-Langmuir current I_{CL} will refer to the current that flows between the vacuum boundary and the virtual cathode when $E_o > 0$ and will refer to the current that flows between the vacuum boundary and probe when $E_o = 0$. The voltage of the probe, the virtual cathode, and the vacuum boundary will be known as V_p, V_v, and V_B, respectively.

It will now help us to consider three cases: $E_o < 0$, $E_o = 0$, and $E_o > 0$. For $E_o < 0$ and $E_o = 0$, there is no virtual cathode, $I_R = 0$ and $I \equiv I_e + I_R = I_e$. Also, when $E_o = 0$, note that $I = I_e = I_{CL}$. In other words,

$$I = I_e \qquad \text{when } E_o < 0 \qquad \text{Eq.*4/5,5}$$

and

$$I = I_e = I_{CL} \qquad \text{when } E_o = 0 \qquad \text{Eq. *4/5,4}$$

where I_{CL} is calculated using V_p and V_B.

When $E_o > 0$, a virtual cathode exists and $I_R \neq 0$. The current arriving at the virtual cathode from the probe $I_e + I_R$ must be equal to the current I_{CL} flowing from the virtual cathode to the vacuum boundary. Hence,

$$I = I_e + I_R = I_{CL} \qquad \text{when } E_o > 0 \qquad \text{Eq. *4/5,1}$$

where I_{CL} is the current flowing from the virtual cathode to the vacuum boundary (i.e. where I_{CL} is calculated using the virtual cathode voltage V_v rather than the probe voltage V_p). When the energy distribution of the emitted electrons can be described by a temperature T,

$$I_R = -I_e\{1-\exp[e(V_v-V_p)/T]\} \qquad \text{Eq. *4/5,2}$$

and eq. *4/5,1 becomes

$$I = I_e \exp[e(V_v-V_p)/T] = I_{CL} \quad \text{when } E_o > 0 \qquad \text{Eq. *4/5,3}$$

Eq. *4/5,3 is not in a particularly useful form because it is in terms of V_v and not V_B.

In cylindrical geometry, the Child-Langmuir law is [Langmuir,I., Blodgett,K., Phys. Rev. 22, 347 (1923)]:

$$I_{CL}(\mathrm{X}10^{-6}\,\mathrm{Amps}) = 14.68\{L(\mathrm{cm})\mathrm{X}[V(\mathrm{volts})]^{3/2}\}/[r(\mathrm{cm})\beta^2] \qquad \text{Eq. *4/5,6}$$

where I_{CL} is the current which flows between an electron emitting cathode and an anode when the electric field at the cathode is zero, V is the voltage between the cathode and anode, L is the length of the cathode, r is the anode radius and β is a constant which depends on the ratio of the anode and emitting cathode radii and in practice has a value close to 1.0 under most circumstances. When a virtual cathode is present $V=V_B-V_V$, $I=I_{CL}$ and eq. *4/5,6 can be rearrange as

$$V_B-V_V = [(r\beta^2/L)(I/14.68)]^{2/3} \qquad \text{Eq. *4/5,7}$$

Taking the natural logarithm of Eq. *4/5,3 and rearranging produces

$$V_V = V_p+(T/e)\ln(I/I_e) \quad \text{when } E_o > 0 \qquad \text{Eq. *4/5,8}$$

We can combine eq.s *4/5,7 and *4/5,8 and write

$$V_p-V_B = (T/e)\ln(I_e/I)-[(r\beta^2/L)(I/14.68)]^{2/3} \quad \text{when } E_o > 0 \qquad \text{Eq. *4/5,9}$$

Eqs. *4/5,5, *4/5,4, and *4/5,9 describe the three cases we have been considering: $E_o<0$, $E_o=0$ and $E_o>0$.

In order to better compare the equations describing the probe in the plasma (eqs. *4/9,1, *4/20,1) with the equations describing the probe in the vacuum (eqs. *4/5,5, *4/5,4 and *4/5,9) note that eq.s *4/5,4 and *4/5,6 can be combined with $V=V_B-V_P$ when $E_o=0$ to yield

$$V_P-V_B = -[(r\beta^2/L)(I/14.68)]^{2/3} \quad \text{when } E_o = 0 \qquad \text{Eq. *4/5,10}$$

Furthermore, from eq. *4/5,1

$$I < I_e \qquad \text{when } E_o > 0 \qquad \text{Eq. *4/5,11}$$

Combining eq. *4/5,11 with eq. *4/5,9 gives

$$V_P-V_B > -[(r\beta^2/L)(I/14.68)]^{2/3} \quad \text{when } E_o > 0,\ I < I_e \qquad \text{Eq. *4/5,12}$$

Hence, from eqs. *4/5,10 and *4/5,12, we conclude that

$$E_o < 0 \qquad \text{when} \quad V_P-V_B < -[(r\beta^2/L)(I/14.68)]^{2/3} \qquad \text{Eq. *4/5,13a}$$

$$E_o = 0 \qquad \text{when} \quad V_P-V_B = -[(r\beta^2/L)(I/14.68)]^{2/3} \qquad \text{Eq. *4/5,13b}$$

and $$E_o > 0 \qquad \text{when} \quad V_P-V_B > -[(r\beta^2/L)(I/14.68)]^{2/3} \qquad \text{Eq. *4/5,13c}$$

Eq. *4/5,13 can be combined with eqs. *4/5,5, *4/5,4 and *4/5,9 to give

$I = I_e$ when $V_p - V_B \leq -[(r\beta^2/L)(I/14.68)]^{2/3}$ Eq.*4/19,1

and $V_p - V_B = (T/e)\ln(I_e/I) - [(r\beta^2/L)(I/14.68)]^{2/3}$

when $V_p - V_B > -[(r\beta^2/L)(I/14.68)]^{2/3}$, $I < I_e$ Eq. *4/19,3

Eqs. *4/19,1 and *4/19,3 can be compared to eqs. *4/9,1 and *4/20,1.

Both probes are biased to the same current I, and I is less than I_e. Hence, it is appropriate to compare eqs. *4/20,1 and *4/19,3 when considering the error due to space charge effects in the presence of plasma gradients. If V_E is defined as

$V_E \equiv (V_{PP} - \phi_P) - (V_{PV} - V_B)$ Eq. *4/20,2

where V_{PP} and V_{PV} refer to the voltage of the probe in the plasma and the probe in the vacuum, respectively, then the maximum error in the electric field measurement E_E is

$E_E = V_E/d$ Eq. *4/20,3

where d is the absolute value of the distance between the probes and a positive value of E_E denotes a electric field that points from the probe in the plasma to the probe in the vacuum. Substituting eqs. *4/20,1 and *4/19,3 into eqs. *4/20,2 and *4/20,3 gives

$V_E = [(r\beta^2/L)(I/14.68)]^{2/3}$ Eq. *4/20,4

and

$E_E = [(r\beta^2/L)(I/14.68)]^{2/3}/d$ Eq. *4/20,5

Note that because $E_E > 0$, space charge adds to an electric field measurement an erroneous electric field that is antiparallel to the density gradient. Also note that eq. *4/20,5 suggests that minimizing I and maximizing L will minimize E_E. However, I is usually proportional to L; so increasing L will not help in general as I also increases, and decreasing I will not help if it is done by decreasing L. I is also usually proportional to the radius of the probe r_p and so minimizing r_p will clearly help. There are of course limits as to how small r_p can be made, namely, a limit as to how thin a probe can be made and a limit as to how small the current bias can be made.

Not all satellite electric field probes are cylindrical; many, if not most, are spherical. The Child-Langmuir law in spherical coordinates is [Langmuir, I., Blodgett, K., Phys. Rev. 23, 49 (1924)]:

$I_{CL}(\mathrm{X}10^{-6}\mathrm{Amps}) = \{29.36[V(\mathrm{volts})]^{3/2}\}/\alpha^2$ Eq. *4/20,6 where α is a function of the radii of the anode and emitting cathode. Some typical values of α^2 are 0.509, 1.022, 2.073, 4.002, and 8.523 which correspond to values of the ratio of anode radius to cathode radius of 1.8, 4.4, 14, 160, and 100000, respectively. The same sort of analysis that lead to eqs. *4/20,4 and *4/20,5 gives

$$V_E = [\alpha^2 I/29.36]^{2/3} \qquad \text{Eq. *4/20,7}$$

and

$$E_E = [\alpha^2 I/29.36]^{2/3}/d \qquad \text{Eq. *4/20,8}$$

Eqs. *4/20,5 and *4/20,8 point out some similarities and differences between cylindrical and spherical probes. In both geometries, making I small is important if E_E is to be kept to a minimum and limitations on the minimum probe size and the minimum current to which the probes can be biased also limit how small I can be made. On the other hand, although each situation in which probes are used must be considered separately, cylindrical geometry has the seeming advantage that E_E can also be reduced by making L large (while keeping I constant by making r_p small).

Lastly, a few comments of clarification concerning I in Eqs. *4/20,5 and *4/20,8. We have been refering to I as the current bias of the probe. This is only an approximation. Plasma gradients can change the current vs. voltage characteristic of a probe in two ways: by changing the space charge in the vicinity of the probe and thereby indirectly affecting the emitted current; and by contributing a collected current to the probe. In the derivation of Eqs. *4/20,5 and *4/20,8, only the plasma gradients effects on the emitted current are included; the plasma gradient effects on collected current are not included. Specifically, as was noted, collected currents are set to zero and ignored in Eqs. *4/9,1 and *4/9,2. Therefore, the I of Eqs. *4/20,5 and *4/20,8 is actually only that fraction of the probe's current bias that is due to emitted current and not the whole current bias.

If, as we have previously assumed, the emitted saturation current is much greater than the collected saturation current, and if, as is often the case with satellite probes, the value of the probe current bias is not close to either of the saturation currents but lies somewhere in between them, then I of Eqs. *4/20,5 and *4/20,8 is approximately equal in value to the current bias of the probe, that is to say, most of the current flowing through the probe in this case is emitted current rather than collected current. Under these circumstances, there is little error in refering to I of Eqs. *4/20,5 and *4/20,8 as the current bias of the probe. The advantage of refering to I in this way is that it simplifies the application of Eqs. *4/20,5 and *4/20,8 to probe data; it is the total current bias of a probe, and not just that part of the current bias which is due to emitted current, that is most often measured.